Outline

1. Introduction
   a) type I and type II superconductors
   b) phase diagram of type II superconductors

2. Basic measurements testing phase diagram
   a) magnetic measurements
      - m-H (T=const)
        critical state models, hysteresis loop in real superconductors: fishtail anomaly, thin films
      - m-T(H=const)
        diamagnetic and Meissner effects, irreversibility line, paramagnetic Meissner effect
      - ac susceptibility
   b) transport and magnetotransport
      - technical details
        measurement geometry, the role of microstructure
      - vortex lattice melting
      - vortex liquid-vortex glass transition
        scaling predictions and methodology
   c) tunneling spectroscopy

3. Superconducting-ferromagnetic heterostructures
   a) exchange (proximity) effects
      - space modulation of the superconducting order parameter
   b) orbital effects
      - local magnetic induction measurements of the magnetic pinning effects

4. Superconductor-insulator transition
**Meissner effect and diamagnetic effect**

**ZFC = diamagnetic (shielding) effect**

1) cooling sample without field
2) measurement during warming

Ideal superconductor: \( B = 0 \rightarrow M = -H \)

\[ M_{ZFC} = - \frac{V}{1-n} H \]

\[ \frac{1}{1-n} \approx \frac{r}{1} \]

\( V \) - sample volume
\( n \) - demagnetization factor

\[ V = \frac{4}{3} \pi r^3 t \]

\[ M \] - magnetization

\[ \mu_0 = 4\pi \times 10^{-7} \text{T.m/A} \]

\( H \) - magnetic field

\( B \) - magnetic induction

****FC = Meissner effect****
measurement during cooling in the field

FC less than ZFC: \( |M_{FC}| < |M_{ZFC}| \)

flux pinning by voids between grains

**G. Xiao et al., PRB 38, 776 (1988)**
Meissner effect and diamagnetic effect

**Phase Diagram**

- $T^*$: irreversibility line
- $T_c$: critical temperature

M. F. Schmidt et al., PRB48 (1993)

**Artifacts:**
- Inability of the signal analysis to properly fit the SQUID output
- Another problem: scan length (homogeneous field !!!)

ZFC: diamagnetic (shielding) effect
FCC: field-cooled with data collected on cooling
FCW: field-cooled with data collected on warming

**SQUID**

- Nb film 500 nm thick
- $H = 1000$ G

J. Deak et al., PRB49 (1994)
Meissner effect and diamagnetic effect

\[ \frac{dB}{dx} = \mu_0 J^c_\text{c}(T); \quad J^c_\text{c}(B) = \text{const}; \quad J^c_\text{c}(T) \text{-- decreasing function of } T; \]

\[ H_0 \rightarrow T_{c2}(H_0) = T \text{ at which } H_{c2}(T) = H_0 = T_c \]

\[ T_{c1}(H_0) = T \text{ at which } H_{c1}(T) = H_0 \]

- **Flux expulsion**
  - \( T_{c2} \):
    - \( \frac{dB}{dx} = 0; \ B(R) = H_0 \)
    - \( T^* \)

- **Flux entering**
  - \( T_{c1} \):
    - \( \frac{dB}{dx} = \text{max}; \ B(R) = 0 \)

\( T^* \) - approximate position of the irreversibility line: temperature below which flux becomes rigidly pinned (vortex glass)

\( T_{vc} \) - no special significance (result of flux trapping)

J. R. Clem and Z. Hao, PRB48 (1993)

J. Deak et al., PRB49 (1994)
Paramagnetic Meissner Effect

S. Riedling et al., PRB49 (1994)

YBa$_2$Cu$_2$O$_7$ single crystal

D.J. Thompson et al., PRL75 (1995)

Wohleben effect:
ceramics,
single crystals,
films,
high-$T_c$,
low-$T_c$

Depends on:
sample geometry,
surface quality,
measurement geometry

Possible origin:
nonuniform field distributions created by
strong flux pinning on the surface layer
Ac susceptibility

\[ B = B_{dc} + b(t) \]

magnetic induction inside the sample:

\[ b(t) = h_0 \cos(2\pi ft) + 4\pi m(t) \]

Fourier expansion of \( m(t) \):

in terms of frequency \( f \) and a complex susceptibility \( \chi_n = \chi_n' + i \chi_n'' \)

\[ m(t) = h_0 \sum_{n=1}^{\infty} \left[ \chi_n' \cos(2\pi ft) + \chi_n'' \sin(2\pi ft) \right] \]

\( \chi_1' \) - a measure of screening

\( \chi_1'' \) - a measure of loss

Onsets in \( \chi_1' \) and in \( |\chi_3| \) coincide, and they mark the onset of screening (onset of fluctuations)

Peak in \( \chi_1'' \): depends on \( f \), on sample shape and geometry, and on sample resistivity - it should not be used to determine the onset of irreversible behavior

J. Deak et al., PRB49, 6270 (1994)
**Transport: measurement geometry**

- **R**: sample resistance; becomes very small in the vicinity of $T_c$
- **$V_1, V_2$**: voltage drops on contacts; become very large at low $T$

**2-point method**

$I$

$U = V_1 + V_2 + IR$

**4-point method**

$I$

$U = V_1 + V_2 + IR$

$IR = \frac{U_+ - U_-}{2}$

**Bridge geometry**

- Hall coefficient measurements on films (photolithography and contact evaporation)
- $R$: $V_1 - V_2$
- $R_H$: $V_3 - V_4$

- Simultaneous measurements of $R$ and $R_H$
- Narrow voltage leads (small misalignment of the Hall contacts)
- Large contact surface area
- Long $R$ bridge

**Square bridge**

- Useful for probing of the directional vortex motion (anisotropy)
- $\mathbf{F}_L = \mathbf{J} \times \mathbf{B} = \mathbf{J} \times \mathbf{n} \Phi_0$
- $F_x = J_y \Phi_0$
- $F_y = J_x \Phi_0$

- $V_y = V_1 - V_2$
- $V_x = V_2 - V_3$
Avoiding surface (edge) barriers for vortices: circulating vortices

D. Lopez et al., PRL82 (1999)

\[
\rho \frac{I}{d} \int_{n}^{n+1} Edr = \ln \left( \frac{r_{n}}{r_{n+1}} \right)
\]

Current density \( J \sim 1/r \)  
Lorentz force \( F_L \sim 1/r \)  
Radial velocity \( v \sim 1/r \)

\[
V_{n,n+1} = \int_{n}^{n+1} Edr = \frac{\rho I}{2\pi d} \ln \left( \frac{r_{n}}{r_{n+1}} \right)
\]

- \( \rho \) - resistivity; \( I \) - current;  
- \( d \) - sample thickness.

Van der Pauw geometry

Arbitrary shape of thickness \( d \) without holes

- \( i_{MN} \) - current  
- \( V_p - V_O \) - potential difference  
- \( i_{NO} \) - current  
- \( V_M - V_P \) - potential difference

\[
\exp \left( -\frac{\pi d}{\rho} R_{MN,OP} \right) + \exp \left( -\frac{\pi d}{\rho} R_{NO,PM} \right) = 1
\]

Hall effect:

- \( i_{MO} \): \( V_P - V_N \to R_{MO,NP} \)  
- \( B \perp \to \Delta R_{MO,NP} \)

$T_{c_R}$ - resistive transition (at $\rho = \rho_N/2$)
$T_{c_X}$ - from ac susceptibility (onset of $\chi_1'$)

Homogeneous bulk superconductor:
$T_{c_R} \sim$ mean field $T_c$
$T_{c_X} \sim$ onset of fluctuations

La$_{1.85}$Sr$_{0.15}$CuO$_4$ films on SrLaAlO$_4$ substrates

In-plane lattice parameters 3.777Å

Pressure studies:
compressive in-plane strain enhances $T_c$

$\rho_N = \rho / \rho_{1.2T_{cR}}$

$T_{cR} \approx T_{cX}$

$\chi_c$ - from ac susceptibility (onset of $\chi_1'$)

Large compressive strain, gradual strain relief

Pressure studies: compressive in-plane strain enhances $T_c$

$T_{cR} \approx T_{cX}$

3D growth

Large tensile strain
Weakly coupled grains

Small compressive or tensile strain

Large tensile strain
Weakly coupled grains

In-plane lattice parameters 3.756Å

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compressive in-plane strain enhances $T_c$
Vortex lattice melting
(1-st order transition)

YBa$_2$Cu$_3$O$_7$ clean
detwinned crystal

Ac-resistance measurements, $f=17\text{ Hz}$

- Agreement with the Lindeman melting
criterion for anisotropic superconductors
  (Blatter, 1992; Beck, 1992)

- Twin boundary pinning at $T_{\text{TB}}$
suppresses the melting transition

W.K.Kwok et al., PRL69 (1992)
Magnetotransport

Vortex lattice melting (1-st order transition)

YBa$_2$Cu$_3$O$_7$ clean detwinned crystal

Ac-resistance measurements, f=17Hz

Thermodynamic evidence of melting in YBa$_2$Cu$_3$O$_7$

U. Welp et al., PRL 76 (96)

W.K. Kwok et al., PRL 69 (1992)

Anisotropy: stack of CuO$_2$ planes
Magnetotransport and thermal conductivity

Vortex lattice melting in URu$_2$Si$_2$

Heavy fermion system:
“hidden order” transition at 17.5K; low carrier density,
exotic superconductivity with d-wave symmetry and $T_c \sim 1.45K$

Thermal conductivity:
no fluctuation correction $\Rightarrow$
gives signature of $T_c (H)$ (mean field)
Niven and Smith, PRB 66 (2002),
Vishveshwara and Fisher, PRB 64 (2001)

- $T_m$: peak positions from $d\rho/dT$
- I-V: $E \sim J^q$, $q=1$ for $T>T_m$
steepest increase of exponent at $T_m$
i.e. transition to non-Ohmic $\rho$

Low T:
mainly electronic contribution
$k/T \sim N(0) v_F l$

$N(0)$ - density of states,
$v_F$ - Fermi velocity,
$l$ - mean free path

$T<T_c$: $N(0) \downarrow$, $l \uparrow$
low $H$: enhancement of $l$
high $H$ - decrease of $N(0)$
Magnetotransport

Vortex liquid-vortex glass transition in disordered systems (continuous)

\[ E = \frac{V}{L} \quad J = \frac{I}{A} \]

Yang et al., PRB 76 (2007): MgB_2 film 100 nm

High T: curves with positive curvature

low J: \( E \sim J \) (Ohmic \( \rho \))

crossover to power-law at large J: \( E \sim J^\alpha \)

I-V: isotherms at \( \sim T_c \)

\[ \rho = 0 \] in the low-J limit

Transition isotherm \( \Rightarrow T_g \)

Highest J: heating of the sample

Low T: negative curvature

Physical quantities must scale with the VG correlation length, $\xi_{VG}$, and a characteristic relaxation time $\tau$, which diverge as $T \to T_g$:

$$\xi_{VG} \propto (T - T_g)^{-\nu} \quad \tau \propto (T - T_g)^{-z\nu}$$

$z$, $\nu$ - dynamic and static critical exponents

Scaling for E-J takes the form:

$$E / \left( J \left| T - T_g \right|^{\nu(z+2+d)} \right) = f_\pm \left( J / \left| T - T_g \right|^{\nu(D-1)} \right)$$

$D$ - dimensionality, $f_\pm$ - two scaling functions, below and above $T_g$.

At $T_g$: $E / (J)_{T=T_g} \approx J^{(z+1)/(D-1)}$

$T>T_g$: $\rho = \frac{dE}{dT} \bigg|_{J \to 0} \propto (T - T_g)^{\nu(z+2+d)}$

Two other variations of scaling:

- in true 2D systems there is no zero resistance at $T>T_g$ (i.e. $T_g=0$): $\rho \propto e^{-(T/T_0)^b}$

- in 2D superconductors at $\mu_0 H = 0$ the vortex-antivortex unbinding transition occurs (Berezinski-Kosterliz-Thouless transition)

Problem: $T_g$ is not uniquely defined

$\Rightarrow$ scaling exponent may vary by a large factor
Vortex liquid-vortex glass transition in disordered systems (continuous)

Strachan et al., PRL 87 (2001): YBaCuO film 220 nm

\[ T_g \] is not uniquely defined \( \rightarrow \) scaling exponent may vary by a large factor
Two isotherms on both sides of $T_g$ (equally spaced away) measured at the same $J$-value should have opposite curvature.

Or, derivatives of the isotherms should approach two different limits below and above $T_g$.

This condition is difficult to fulfill experimentally!!
The tunneling conductance is related to local DOS:

\[ I_{ns} \sim |M|^2 \mathcal{N}_n \int_{-\infty}^{\infty} \mathcal{N}_s(E) [f(E) - f(E+eV)] \, dE \]

\[ \mathcal{N}_n, \mathcal{N}_s - \text{density of states}, \]
\[ f(E) - \text{Fermi distribution function}, \]
\[ M - \text{tunneling matrix element} \]

**Andreev reflection (small barrier)**

Electron is reflected from the S/N interface as a hole, leaving Cooper pair on the S side \( \Rightarrow \) conductance is twice the normal-state value

**Sharvin point contact**

G. Deutscher, RMP 77 (2005)

**Z - barrier parameter:**

Phase-sensitive effect
Andreev reflection at the S/F interface

Andreev reflection is cut off for large spin polarization

Point-Contact Andreev Reflection (PCAR) spectroscopy

Crossed Andreev reflection

Spin-filter: the conduction is possible when the F leads are polarized antiparallel

Thin Fe wires: magnetic anisotropy

$\Delta R$: difference between the parallel and antiparallel alignment of Fe wires

$\Delta$: difference between the parallel and antiparallel alignment of Fe wires
Breaking of the singlet Cooper pair

Nanoscale range of layer thicknesses

Exchange mechanism:
- Aligning spins in the same direction occurs in the intermediate vicinity of the S/F interfaces (proximity effect).

Orbital mechanism:
- The interaction of the superconducting order parameter with a vector potential $A$ of the magnetic field.
- Long-range effect, may act across thin insulating barrier which cuts off proximity effect.

Standard BCS pairing:
- $\langle k \uparrow, -k \downarrow \rangle$ S=0 (spin singlet)
- S=1 (spin triplet)

S/F structure with both exchange and orbital effects

S/F structure with exclusively orbital effects

A.A. Golubov et al., RMP 76 (2005); A.I. Buzdin, RMP 77 ('05); F. S. Bergeret, RMP 77 ('05), K.B. Efetov et al., in Magnetic Heterostructures (Springer, 2006)

**Exchange interactions**

Proximity effect at the S/F interface - interaction with the exchange field $h_{ex}$:

$k_F \rightarrow k_1 = k_F + \delta k_F$

$k_F \rightarrow k_2 = -k_F + \delta k_F$

$\delta k_F = \frac{\mu_B h_{ex}}{v_F}$

Cooper pair acquires finite center-of-mass momentum, $2 \delta k_F$

Cooper pairs penetrate through interface

$\xi_n = \frac{\hbar D}{k_B T} \approx 100 \text{nm}$

$\xi_s = \frac{\hbar D}{k_B T_{Curie}} \approx 1 \text{nm}$

Cooper pair acquires finite center-of-mass momentum, $2 \delta k_F$

π-shift of the phase of superconducting order parameter

- S/F bilayer and S/F/S structures

DOS modulation in F layer

Kontos et al., PRL86 (2001)

Zdravkov et al., PRL97 (2006)

$\xi = \frac{\hbar D}{k_B T_{Curie}} \approx 1 \text{nm}$

CuNi on Nb (grown by magnetron sputtering)

Oscillatory $T_c(d_F)$ – multilayers ('95), reentrant $T_c(d_F)$ – bilayers (2006)

Silicon cap and buffer layer

mK measurements

Planar tunneling spectroscopy differential conductance measurements

Kontos et al., PRL86 (2001)
Exchange interactions

Interplay of proximity effect and inhomogeneous magnetization of the F-layer (domain structure)

Enhancement of $T_c$ (weaker exchange)
Generation of triplet supercurrent (long-range proximity effect)


Spin-orientation dependent superconductivity in F/S/F structures, $d_S \leq \xi$

Superconducting spin valve
(transparent interface):


$$T_c^{AP} > T_c^P$$

J. Y. Gu et al., PRL (2002)

Inverse spin switch (insulating interface or F-layer highly spin polarized $\rightarrow$ spin accumulation):

$$T_c^{AP} < T_c^P$$


Experiments: A. Yu. Rusanov et al., PRB (2006); A. Singh et al., PRB (2007); V. Peña et al., PRL (2005); P. Przyslupski et al., SST (2005).

Unclear role of orbital effects