Critical quantum thermometry and its feasibility in spin systems

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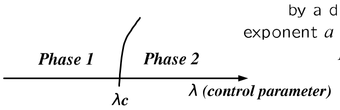
Abstract: In this work [1], we study temperature sensing with finite-sized strongly correlated systems exhibiting quantum phase transitions. We use the quantum Fisher information (QFI) approach to quantify the sensitivity in the temperature estimation, and apply a finite-size scaling framework to link this sensitivity to critical exponents of the system around critical points. We numerically calculate the QFI around the critical points for two experimentally-realizable systems: the spin-1 Bose-Einstein condensate and the spin-chain Heisenberg XX model in the presence of an external magnetic field. Our results confirm finite-size scaling properties of the QFI. Furthermore, we discuss experimentally-accessible observables that (nearly) saturate the QFI at the critical points for these two systems.

[1] Enes Aybar, Artur Niezgoda, Safoura S. Mirkhalaf, Morgan W. Mitchell, Daniel Benedicto Orenes, Emilia Witkowska, Quantum 6, 808 (2022)

Criticality



In the thermodynamic limit ($N\to\infty$, $L^d\to\infty$, $N/L^d=$ const.) the QPT is characterized by a diverging power law behavior of a physical quantity $A\sim\varepsilon^a$ given by a critical exponent a quantifying how rapidly A changes at λ_c. If the size of the system (L and N) is finite then the change of A is an analytic function of ε and a regular function f_a such that $A\sim L^{-a/\nu} f_a(L^{1/\nu})$ with the constraint $f_a(0)\neq 0$.

Finite-size scaling of QFI

When the temperature T is low, i.e, $T\sim<\Delta_g$, the main contribution to the QFI comes from the terms containing T/Δ_g while the contribution of $T/\Delta_n>1$ is assumed to be negligible, then

$$F_Q = \Delta_g^{-2}\left(\frac{\Delta_g}{T}\right)^4 \frac{1}{4\cosh^2\left[\frac{\Delta_g}{2T}\right]} \sim N^{2z/d}$$

for fixed $\varepsilon N^{1/(\nu d)}$ and T/Δ_g and non-degenerate systems

$$F_Q^{\max} \approx 4.53\Delta_g^{-2} \text{ for } T/\Delta_g \approx 0.24$$

Finite-size scaling of SNR

The relative estimation precision (shot-noise ratio SNR) is dimensionless itself and it can be related to the QFI, and hence

$$\frac{T}{\sqrt{\delta^2 T}} = \left(\frac{\Delta_g}{T}\right)\frac{1}{4\cosh\left[\frac{\Delta_g}{2T}\right]}$$

unlike the QFI, the SNR does not exhibit the scaling with the total number of particles for fixed values of $\varepsilon N^{1/(\nu d)}$ and T/Δ_g and non-degenerate systems

Local Quantum Thermometry

Purpose: estimation of temperature T of systems that exhibit continuous quantum phase transition (QPT).

In the finite-size system having the total number of particles N and described by a parametrized Hamiltonian H(λ), the behaviour of physical quantities are regular at critical points and can be described by analytic functions that are subject to a universal finite-size scaling. At small but finite temperatures, Gibbs states

$$\hat\rho(T,\lambda) = \sum_{n=0}^{\mathcal{N}} \frac{e^{-\Delta_n(\lambda)/T}}{Z}|\psi_n\rangle\langle\psi_n|$$

where $\hat H(\lambda)|\psi_n\rangle = E_n(\lambda)|\psi_n\rangle$ $\Delta_n(\lambda) = E_n(\lambda) - E_0(\lambda)$

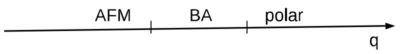
can inherit signatures of quantum critical behavior that, as in the zero-temperature case, enhance sensitivity in parameter estimation around the QPT. The resulting fluctuations of temperature in terms of the mean squared error of the corresponding estimator $\delta^2 T \equiv \langle (T_{\rm est}-T)^2\rangle$ is subject to the Cramér-Rao lower bound

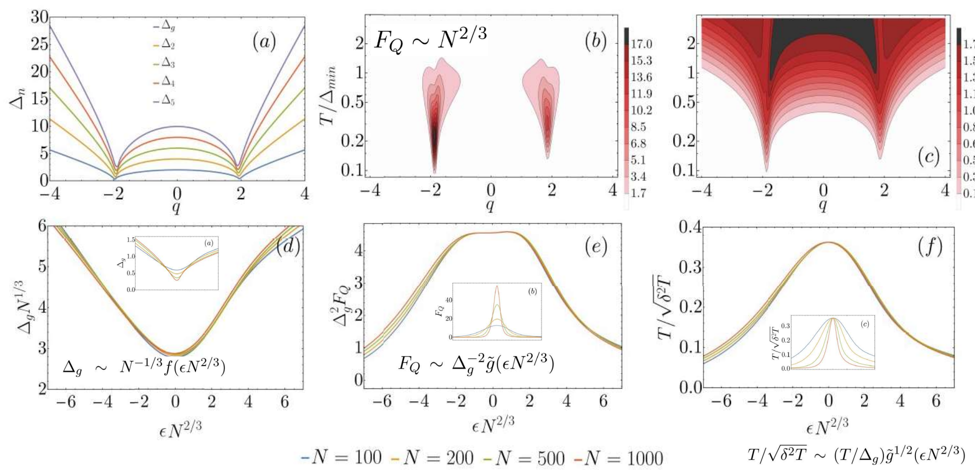
$$F_Q(T,\lambda) = \frac{\Delta^2 \hat H(T,\lambda)}{T^4}$$

where $F_Q(T)$ is the quantum Fisher indfornation (QFI).

Spin-1 BEC

$$\hat H_1 = -\frac{c}{2N}\hat J^2 - q_z \hat N_0$$





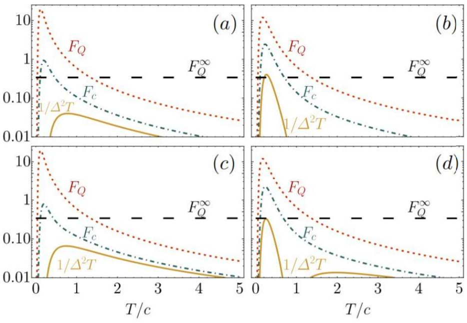
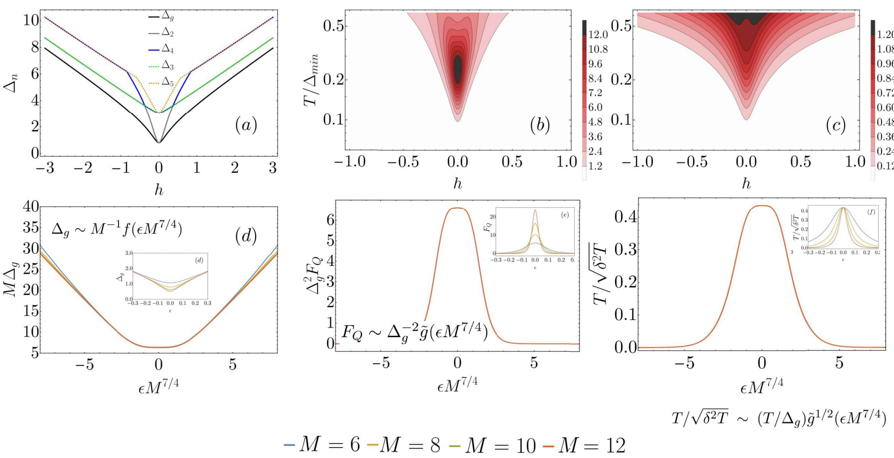


Figure 3: Numerical results showing the temperature dependence of c^2F_Q (red dashed line), c^2F_c (green dot-dashed) and $c^2/\Delta^2 T$ (orange solid) for $\hat J_\perp^2$ (a) and (b), and $\hat N_0$ (c) and (d) with $N=200$ at the critical point $q_z/c=-1.869$ (a),(c), and a near-critical point $q_z/c=-1.8$ (b),(d). The black-dashed lines indicate $\max_T F_Q^*(q_z,T)$ to give a reference level.

Spin-1/2 XX system

$$\hat H_{1/2} = -4J\sum_{j=1}^{M}\left(\hat s_j^x \hat s_{j+1}^x + \hat s_j^y \hat s_{j+1}^y\right) + 2h_x\sum_{j=1}^{M}\hat s_j^x$$



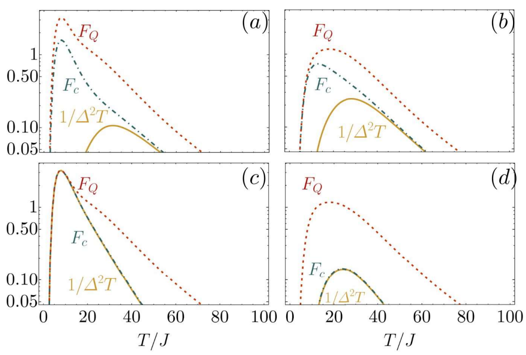


Figure 4: Numerical results showing J^2F_Q (red dashed line), J^2F_c (green dot-dashed) and $J^2/\Delta^2 T$ (orange solid) with $M=4$ as a function of T/J for $h_x/J=0$ (a),(c), and $h_x/J=0.5$ (b),(d) when $\hat A=\hat S_x^2$ (a),(b), and $\hat A=\hat S_z^2$ (c),(d).